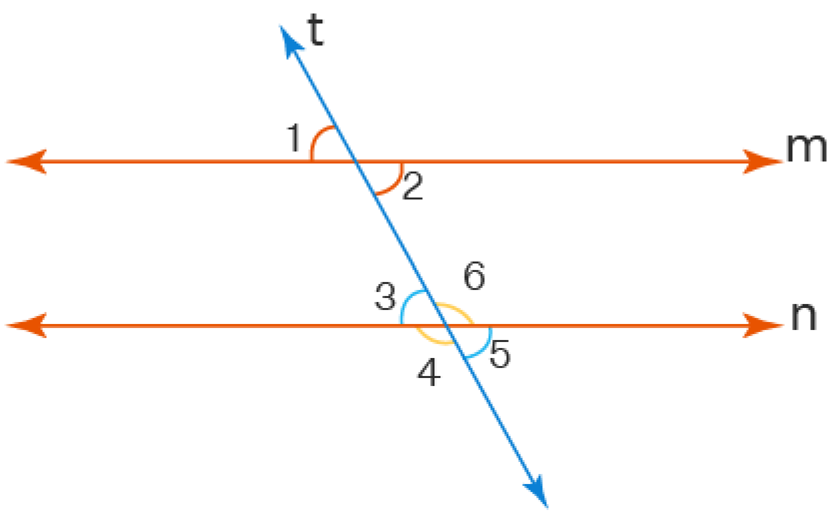
Parallel and Perpendicular Lines in Coordinate Geometry

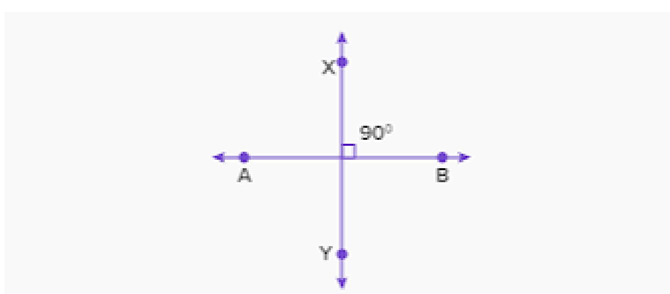
Parallel Lines:

- Imagine train tracks - never meet, always the same distance apart.
- Same slope (m), different y-intercepts (b) in their equations (y = mx + b).
- Graphs are always parallel.



Perpendicular Lines:

- Intersect at a 90° angle (like a corner).
- Slopes (m1 & m2) are negative reciprocals of each other (m1 * m2 = -1).
- Example: Slope 1 (m1 = 8) has a perpendicular line with slope -1 (m2 = -1/8).



Key Takeaway:

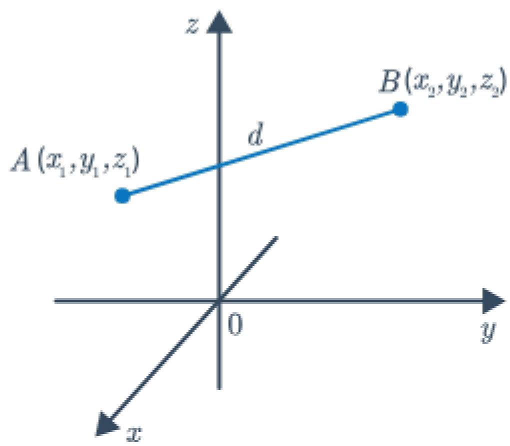
Same slope = Parallel lines

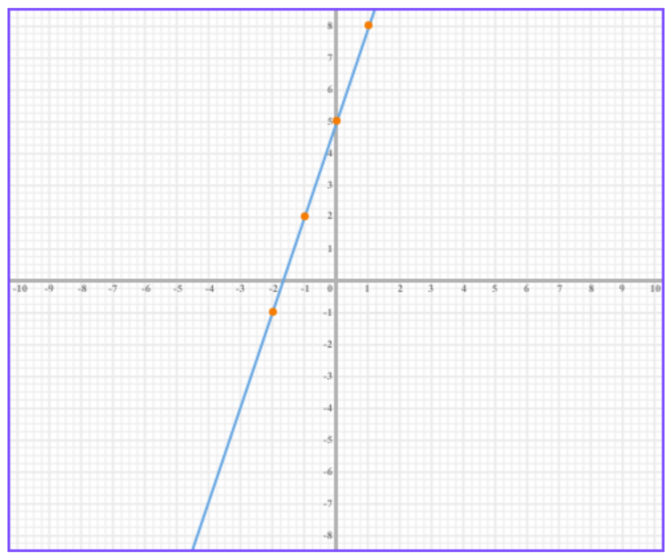
Negative reciprocal slopes = Perpendicular lines

Parallel and Perpendicular Lines in 3D Coordinate Geometry

Lines in 3D:

Unlike 2D, lines in 3D space are defined not just by slope but by direction. We represent this direction with a direction vector.





Distance Formula:

2D Distance: We can find the distance between two points (x1, y1) and (x2, y2) using the distance formula:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3D Distance: Similarly, for points (x1, y1, z1) and (x2, y2, z2) in 3D space:

$$\sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$$

Key Points:

- Direction vectors are crucial for analyzing lines in 3D.
- Slopes (in 2D sense) don't directly apply to 3D lines, but the concept of negative reciprocals for perpendicular lines extends to their direction vectors.
- Distance formulas provide a way to calculate separation between points in both 2D and 3D space.

Distance from a Point to a Line in 2D and 3D

In both 2D and 3D space, the distance between a point and a line refers to the shortest distance between the point and any point on the infinite line. This distance is measured by a line segment perpendicular to the given line, connecting the point to the line.

2D Distance:

We can find the distance between a point (P: x_0, y_0) and a line (represented by the equation Ax + By + C = 0) using the following formula:

$$\frac{\left|Ax + By + c\right|}{\sqrt{A^2 + B^2}}$$

The perpendicular bisector of the line segment AB is perpendicular to AB and passes through the midpoint of AB as shown in the figure:

So, to find the equation of the perpendicular bisector of the segment AB, we need the coordinates of the midpoint of AB and the gradient of AB so that we can use the following property to find the gradient of the perpendicular of the segment:

Perpendicular Bisectors in 2D and 3D Coordinate Geometry

Perpendicular Bisector:

- A perpendicular bisector of a line segment is a line that:
- Intersects the original line segment at a 90° angle (perpendicular).
- Divides the line segment into two segments with equal lengths.

2D Perpendicular Bisector:

Midpoint Formula:

We can find the midpoint (M) of a line segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ using the midpoint formula:

$Mx = (x_1 + x_2) / 2$
$My = (y_1 + y_2) / 2$

Slope and Direction:

- The perpendicular bisector will have a slope that is the negative reciprocal of the original line segment's slope (if it has one).
- If the original line has an infinite slope (vertical), the bisector will have a slope of 0 (horizontal).
- If the original line has a slope of 0 (horizontal), the bisector will have an infinite slope (vertical).

Equation of Bisector:

Once you have the midpoint (M) and the concept of slope (or its special cases), you can use the point-slope form of linear equations to find the equation of the perpendicular bisector.